★ *GrahamFest* ★

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Fermion Mass Textures:
From Family Symmetries to ... F-theory

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$I\omega\alpha\nu\nu\iota\nu\alpha$

GREECE

A: OXFORD 1994-1995:

1. *Neutrino Masses From Gauge Symmetries*
H. Dreiner, G.K.L., S. Lola, G.G.Ross, C. Scheich

2. *Heavy Neutrino Threshold Effects in Low Energy Phenomenology*
G.K.L., S. Lola, G.G.Ross

★ Hierarchical structure of Fermion mass matrices

⇕

U(1) Underlying symmetry (*Ibanez & Ross 1994*)

★ Non zero Neutrino masses

⇕

MSSM is extended to include right-handed neutrinos

	Q_i	u_i^c	d_i^c	L_i	e_i^c	ν_i^c	H_1	H_2
$\mathbf{Q_{U(1)}}$:	α_i	α_i	α_i	a_i	a_i	a_i	$-2\alpha_1$	$-2\alpha_1$

Table 1: SM representations and their $U(1)$ charges

This generated Dirac Mass Textures organised by

$U(1)$- *'family' symmetry*

$$M^{Dirac}_{\nu\nu^c} = \begin{pmatrix} \lambda_{11}\bar{\epsilon}^{|2+6a-2b|} & \lambda_{12}\bar{\epsilon}^{3|a|} & \lambda_{13}\bar{\epsilon}^{|1+3a-b|} \\ \lambda_{21}\bar{\epsilon}^{3|a|} & \lambda_{22}\bar{\epsilon}^{2|1-b|} & \lambda_{23}\epsilon^{|1-b|} \\ \lambda_{31}\bar{\epsilon}^{|1+3a-b|} & \lambda_{32}\epsilon^{|1-b|} & 1 \end{pmatrix} m_\tau$$

- $\bar{\epsilon} \sim \frac{\langle\theta\rangle}{M} \sim \mathcal{O}(10^{-1})$ (θ=*familon*)
- a, b functions of α_i, a_i
- λ_{ij} : unknown $\mathcal{O}(1)$ coefficients

Heavy Majorana Mass Matrix

ν^c's can get masses from terms

$$\langle\phi\rangle\,\nu^c\,\nu^c, \quad \text{with } \phi \sim \tilde{\bar{\nu}}^c\,\tilde{\bar{\nu}}^c$$

Structure of the resulting M_{ν^c} depends on choice of bilinear...

A possible choice...

$$M_{\nu^c\nu^c} \sim \begin{pmatrix} \lambda_{11}\,\bar{\epsilon}^8 & \lambda_{12}\,\bar{\epsilon}^3 & \lambda_{13}\bar{\epsilon}^4 \\ \lambda_{21}\bar{\epsilon}^3 & \lambda_{22}\bar{\epsilon}^2 & \lambda_{23}\epsilon \\ \lambda_{31}\bar{\epsilon}^4 & \lambda_{32}\epsilon & 1 \end{pmatrix} M_0$$

$\cdots + \cdots$ another five discrete cases...

Low Energy Effective neutrino matrix

$$\Downarrow$$

$$\boxed{m_\nu^{eff.} \sim M^D_{\nu\nu^c}\,(M_{\nu^c\nu^c})^{-1}(M^D_{\nu\nu^c})^T}$$

....

$\Rightarrow$ Results are encouraging but depend on Unknown Coefficients

$$\lambda_{ij}^{Dirac},\quad \lambda_{ij}^{Maj.}\quad \sim \mathcal{O}(1)???$$

B: CERN: 2009

Intersection of two corridors, three floors and more...
(F-theory: Ideal place to build Yukawa couplings...)

F-THEORY PHENOMENOLOGY

2010-2011:

1. *Yukawa couplings and fermion mass structure in F-theory GUTs.*
G.K.L., G.G.Ross, JHEP 1102 (2011) 108

2. *Family symmetries in F-theory GUTs.*
S.F. King, G.K.L., G.G.Ross, Nucl.Phys. B838 (2010) 119-135

3. *Towards a Realistic F-theory GUT.*
J.C. Callaghan, S.F. King, G.K.L., G.G.Ross, arXiv:1109.1399

TASKS ...

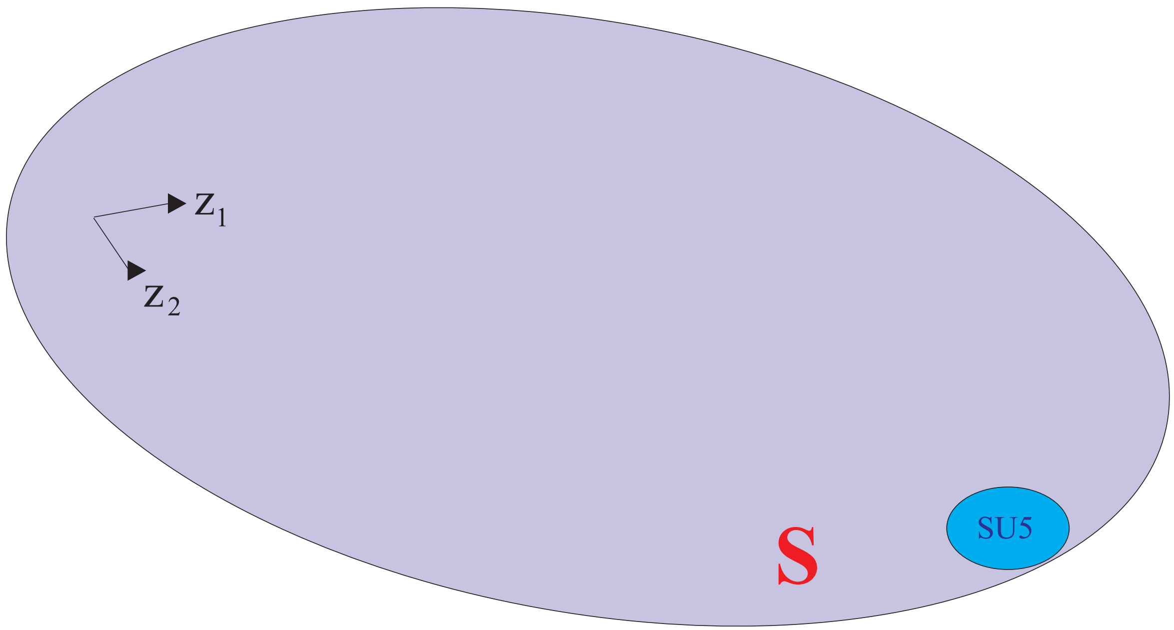
- ... show that 'family' symmetries naturally incorporated in $\mathcal{F}$-Theory GUTs

- ... propose a way to calculate these 'unknown' coefficients

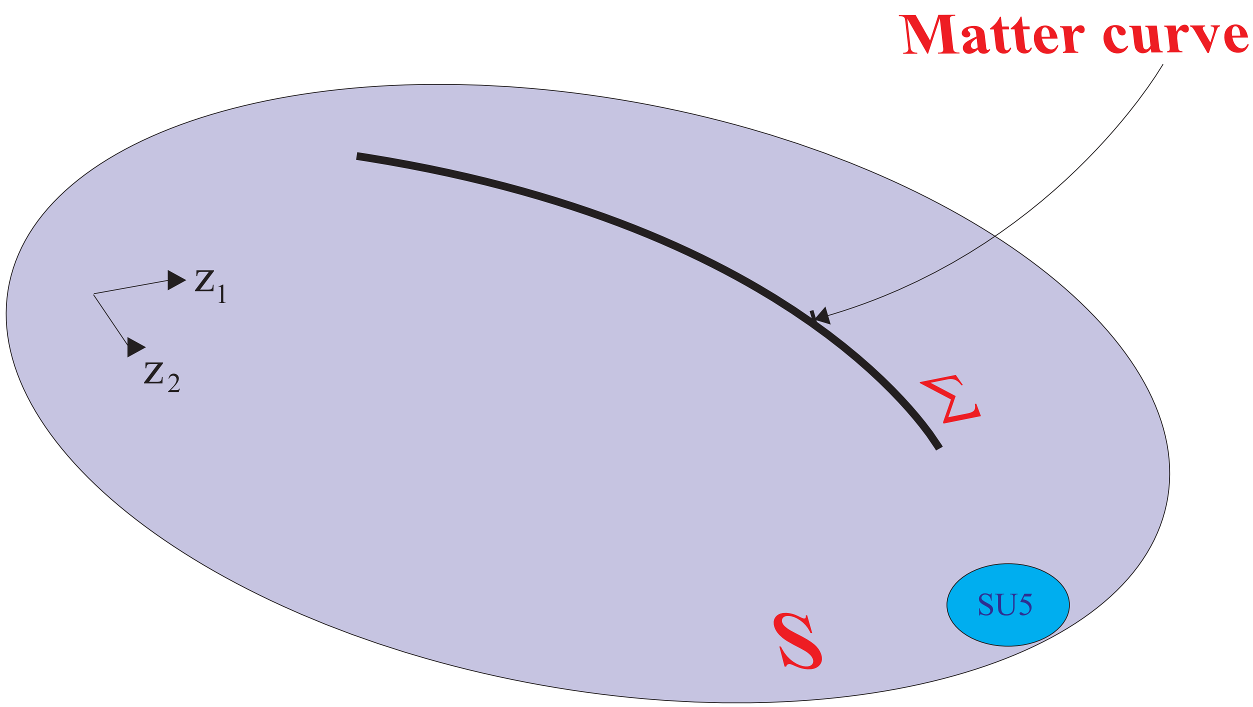
$$\lambda_{ij} \propto \int \psi_{f_i} \psi_{f_j} \phi_H$$

in **F-theory:**

7-branes wrap certain class of *'internal'* **2-complex dim.** surface **S** associated to gauge group G_S (*here taken to be* $SU(5)$)

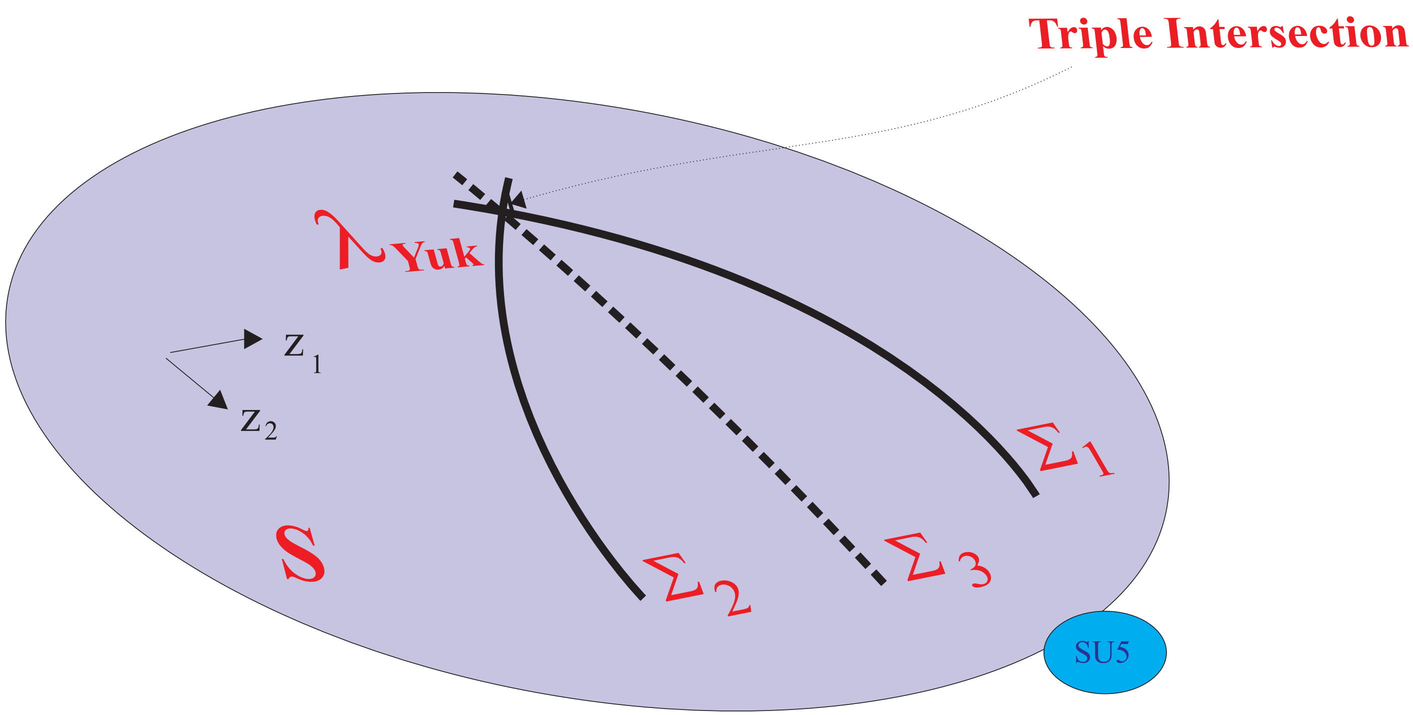


Matter resides along intersections with other 7-branes...

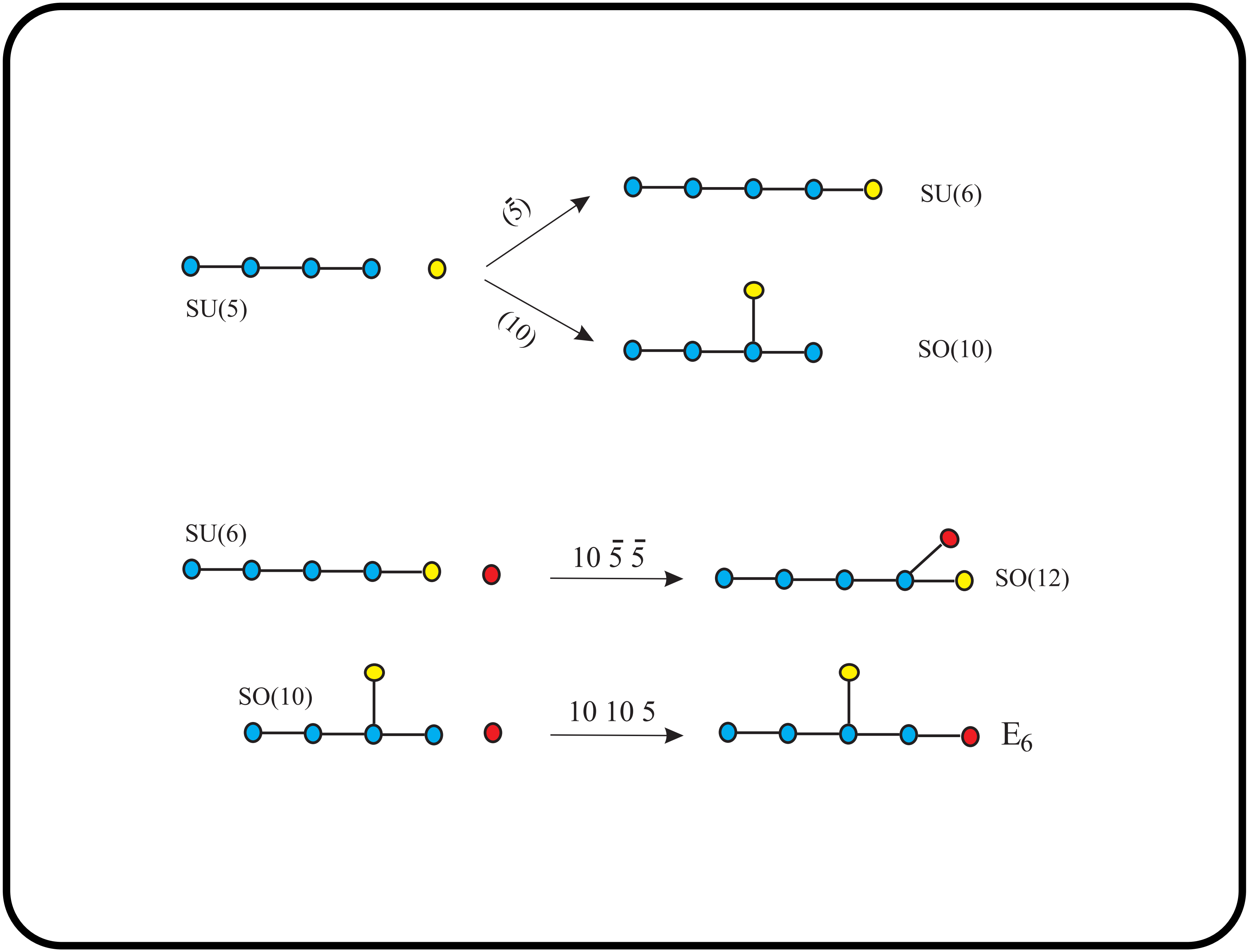


Along a matter curve Σ gauge symmetry is enhanced...

Yukawa couplings



gauge symmetry ... further ... enhanced!


SU(5)
($\bar{5}$)
(10)
SU(6)
SO(10)
SU(6)
10 $\bar{5}$ $\bar{5}$
SO(12)
SO(10)
10 10 5
E_6

$G_S = SU(5)$: Singularity enhancement:

- Matter curves accommodating $\bar{\mathbf{5}}$ are associated with $SU(6)$

$$\Sigma_{\bar{5}} = S \cap S_{\bar{5}} \quad \Rightarrow \quad SU(5) \to SU(6)$$

$$\mathrm{ad}_{SU_6} = 35 \quad \to \quad 24_0 + 1_0 + 5_6 + \bar{5}_{-6}$$

- Matter curves accommodating $\mathbf{10}$ are associated with $SO(10)$

$$\Sigma_{10} = S \cap S_{10} \quad \Rightarrow \quad SU(5) \to SO(10)$$

$$\mathrm{ad}_{SO_{10}} = 45 \quad \to \quad 24_0 + 1_0 + 10_4 + \overline{10}_{-4}$$

- Further enhancement in triple intersections→ **Yukawas**:

$$SO(10) \equiv E_5 \quad \Rightarrow \quad E_6 \to \mathbf{10}\cdot\mathbf{10}\cdot\mathbf{5}$$

$$SU(6) \quad \Rightarrow \quad SO(12) \to \mathbf{10}\cdot\bar{\mathbf{5}}\cdot\bar{\mathbf{5}}$$

★ Zero-modes Equations

Equs of motion from $d = 8$ effective action

$\Downarrow$

D.E.s for **zero modes**:

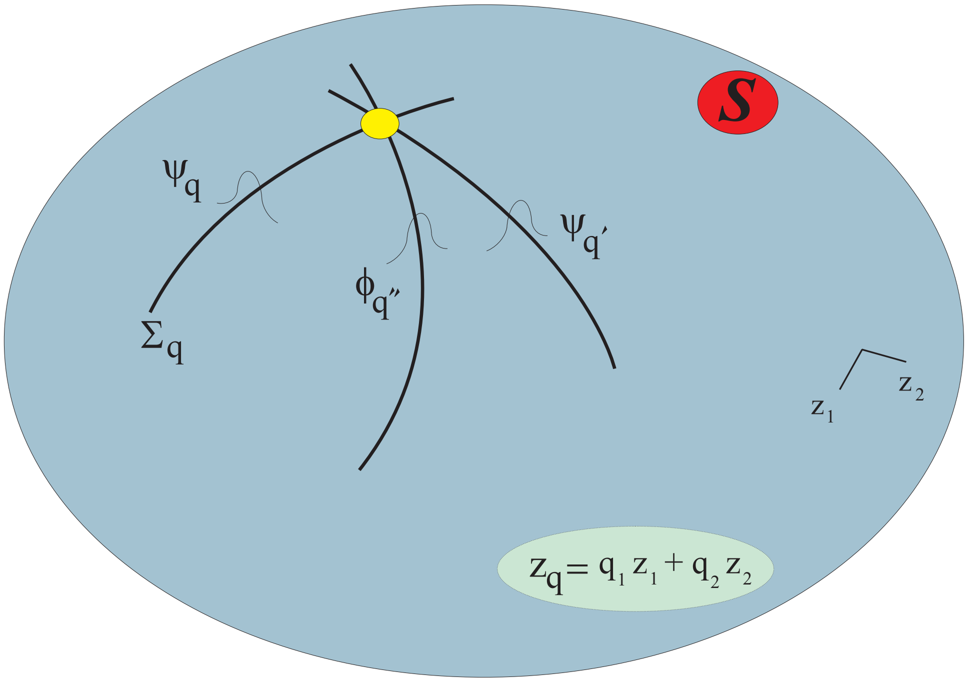
(*Font & Ibanez 2009, Heckman et al 2009* *)

- Solution: Gaussian profile for zero mode wavefunction:

$$\psi \propto e^{\{-m^2 q|\cos\theta z_1 + \sin\theta z_2|^2\}} \tag{1}$$

with $q = \sqrt{q_1^2 + q_2^2}$ *and* $\tan\theta = q_2/q_1$.

(* see also: *Aparicio, Font, Ibanez, Marchesano, 1104.2609*)



Depiction of the wavefuctions ψ_i and ϕ along the matter curves...

Trilinear Yukawa coupling Integral:

- Computation in terms of overlapping wavefunction integrals

$$\lambda_{ij} = \frac{M_*^4}{(2\pi)^2} \int_S \psi_i \cdot \psi_j \cdot \phi \, dz_1 \wedge d\bar{z}_1 \wedge dz_2 \wedge d\bar{z}_2$$

$$\text{Assuming}: \left\{ \begin{array}{l} M_* \sim m \\ M_*^2 R^2 \sim a_G \end{array} \right\} \Rightarrow$$

$$\Rightarrow \lambda_{ij} = \frac{4\sqrt{\pi \, a_G}^{3/2}}{q + q' + q''} \frac{(q q' q'')^{3/2}}{(q_1 q_2' - q_1' q_2)^2}$$

Application to **F-**$SU(5)$★

$E_8 \to$ Highest symmetry in Elliptic Fibration:

$$E_8 \supset SU(5)_{GUT} \times U(5)_\perp \supset SU(5)_{GUT} \times \mathbf{U(1)^4}$$

$\mathbf{U(1)}$'s $\Rightarrow$ family symmetries
Some of the $\mathbf{U(1)}$'s are related by monodromies $\Rightarrow$
...identifying directions t_i in the $SU(5)_\perp$ Cartan subalgebra:

$$Q_t = \mathrm{diag}\{t_1, t_2, t_3, t_4, t_5\}$$

t_i subject to traceless condition: $t_1 + t_2 + t_3 + t_4 + t_5 = 0$.
Z_2 monodromy ($t_1 \leftrightarrow t_2$) gauge symmetry reduces to:

$$\mathbf{SU(5) \times U(1)^3}$$

★*Dudas-Palti, 0912.0853*; *King, GKL, Ross, 1005.1025.*

Fermion Mass Textures

Two ways to obtain Fermion Mass Hierarchy in F-theory

- All families on the same curve(s) ($\Sigma_{10}, \Sigma_{\bar{5}}$)

  Flux corrections $\Rightarrow$ Hierarchy...

- Families assigned on different matter curves ($\Sigma_{10}^{1,2,3}, \Sigma_{\bar{5}}^{1,2,3}$)

  Monodromy $\rightarrow$ Rank one mass matrices at tree level.
  Hierarchy organised by $U(1)$**'s** from underlying E_8 via:
  Singlet vevs $\langle\theta_{ij}\rangle$

Field	$SU(5) \times SU(5)_\perp$ **Rep.**	$\mathbf{t_i}$ direction	R-parity
Q_3, U_3^c, l_3^c	$(10, 5)$	$t_{1,2}$	$-$
Q_2, U_2^c, l_2^c	$(10, 5)$	t_4	$-$
Q_1, U_1^c, l_1^c	$(10, 5)$	t_3	$-$
D_3^c, L_3	$(\overline{5}, 10)$	$t_{1,2} + t_4$	$-$
D_2^c, L_2	$(\overline{5}, 10)$	$t_{1,2} + t_3$	$-$
D_1^c, L_1	$(\overline{5}, 10)$	$t_3 + t_4$	$-$
H_u	$(5, \overline{10})$	$-t_1 - t_2$	$+$
H_d	$(\overline{5}, 10)$	$t_3 + t_5$	$+$
θ_{ij}	$(1, 24)$	$t_i - t_j$	$+$
θ'_{ij}	$(1, 24)$	$t_i - t_j$	$-$

Choice: $\langle\theta_{14}\rangle \cdot \langle\theta_{43}\rangle \neq 0$

- **Rank one** Quark mass matrices (*Dudas-Palti,arXiv:0912.0853*)

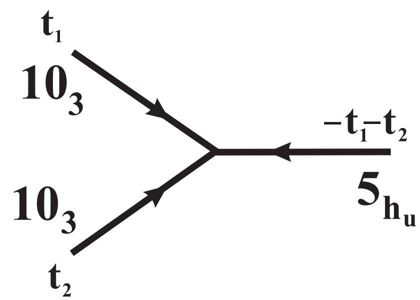
$$M_d = \begin{pmatrix} \lambda^d_{11}\theta^2_{14}\theta^2_{43} & \lambda^d_{12}\theta_{14}\theta^2_{43} & \lambda^d_{13}\theta_{14}\theta_{43} \\ \lambda^d_{21}\theta^2_{14}\theta_{43} & \lambda^d_{22}\theta_{14}\theta_{43} & \lambda^d_{23}\theta_{14} \\ \lambda^d_{31}\theta_{14}\theta_{43} & \lambda^d_{32}\theta_{43} & 1 \times \lambda^d_{33} \end{pmatrix} v_b, \qquad (2)$$

$$M^u = \begin{pmatrix} \lambda^u_{11}\theta^2_{14}\theta^2_{43} & \lambda^u_{12}\theta^2_{14}\theta_{43} & \lambda^u_{13}\theta_{14}\theta_{43} \\ \lambda^u_{21}\theta^2_{14}\theta_{43} & \lambda^u_{22}\theta^2_{14} & \lambda^u_{23}\theta_{14} \\ \lambda^u_{31}\theta_{14}\theta_{43} & \lambda^u_{32}\theta_{14} & 1 \times \lambda^u_{33} \end{pmatrix} v_u \qquad (3)$$

- λ_{ij} computed from overlapping integrals ... *expected of* $\mathcal{O}(1)$.

Are λ_{ij} really $\sim \mathcal{O}(1)$???

Computing $\lambda_{33} \equiv \lambda_{top}$**:** Define 'vector basis': $|t_i >_j = \delta_{ij}$ $(i.e. < t_1| = \{1,0,0,0,0\},\ etc)$



Define Locally the set of orthonormal operators Q_i:

$$Q_1 = \frac{1}{\sqrt{30}}\{3,3,-2,-2,-2\}$$

$$Q_2 = \frac{1}{\sqrt{2}}\{1,-1,0,0,0\},$$

$$Q_3 = \frac{1}{\sqrt{2}}\{0,0,1,0,-1\}$$

$$Q_4 = \frac{1}{2}\{0,0,1,-2,1\}$$

Vertex states $|t_{1,2}>, |-t_1-t_2>$ of top coupling are annihilated by:

$$Q_{3,4}\,|t_{1,2}> = 0$$

while, acting by $Q_{1,2}$:

$$\{q_1, q_2\} = \left\{\sqrt{\frac{3}{10}}, \frac{1}{\sqrt{2}}\right\},\ \{q_1', q_2'\} = \left\{\sqrt{\frac{3}{10}}, -\frac{1}{\sqrt{2}}\right\}$$

Substitution to the overlapping integral:

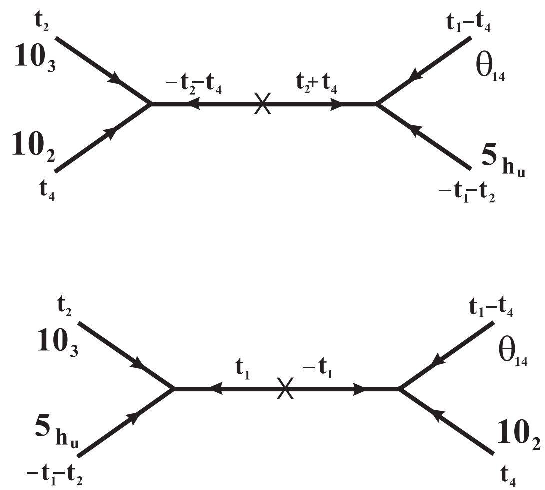
$$\lambda_{top} = 0.31 \times \left(\frac{a_G}{a_{G_0}}\right)^{\frac{3}{4}}, \quad a_{G_0} = \frac{1}{24}$$

Computing higher dimensional couplings

Example:

$$\lambda^u_{23}\, 10_3\, 10_2\, 5_{h_u}\theta_{14} \;\to\; m_{23}\, Q_2\, u^c_3$$

with exchange of massive messenger states



KK-mode wavefunction:

$$\psi_1 \sim f\, e^{-q_1 m^2 \xi\, |z_1|^2}, \;\; \xi < 1$$

♦ **Left** vertex of U_{23}-graphs:

$$10_i 10_j 5_{KK} \; : \; \{t_i\}_{0-mode} + \{t_j\}_{0-mode} \to \{-t_i - t_j\}_{KK}$$

Calculation of the overlapping integral:

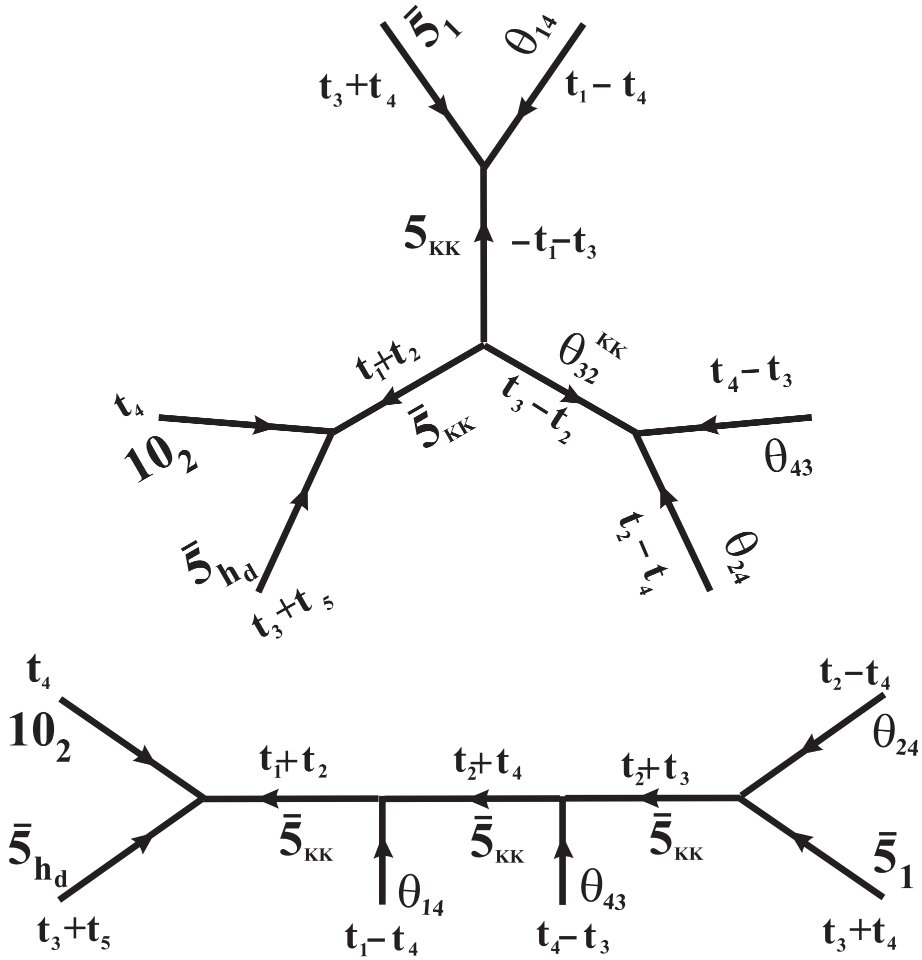
$$I_a(\xi) = \frac{8\sqrt{2\pi\,\xi}}{3 \cdot 5^{3/4}\left(4\xi + \sqrt{6}\right)}$$

♦ **Right** vertices ($\overline{10} \cdot 10 \cdot 1$ and $\overline{5} \cdot 5 \cdot 1$) :

$$I_x(\xi) = \frac{8\sqrt{10\pi\,\xi}}{3 \cdot 3^{3/4}\left(4\sqrt{5}\xi + 5\sqrt{2}\right)}, \; I_y = \frac{2 \cdot 3^{3/4}\sqrt{10\pi}\sqrt{\xi}}{7\left(\left(5 + \sqrt{15}\right)\xi + \sqrt{15}\right)}$$

U_{23} - **Yukawa Coupling:**

$$\boxed{\lambda^u_{23}(\xi) = I_a(\xi) \cdot \left(I_x(\xi) + I_y(\xi)\right)}$$



Representative graphs for λ^b_{21} Yukawa coupling.

Results: (simplified case $\xi = 1$)

$$M_d = \begin{pmatrix} 0.12\,\theta_{14}^2\theta_{43}^2 & 0.11\,\theta_{14}\theta_{43}^2 & 0.18\,\theta_{14}\,\theta_{43} \\ 0.14\,\theta_{14}^2\theta_{43} & 0.16\,\theta_{14}\theta_{43} & 0.20\,\theta_{14} \\ 0.09\,\theta_{14}\theta_{43} & 0.17\,\theta_{43} & 0.29 \end{pmatrix}$$

$$M_u = \begin{pmatrix} 0.09\,\theta_{14}^2\theta_{43}^2 & 0.22\,\theta_{14}^2\theta_{43} & 0.16\,\theta_{14}\theta_{43} \\ 0.22\,\theta_{14}^2\theta_{43} & 0.18\,\theta_{14}^2 & 0.22\,\theta_{14} \\ 0.16\,\theta_{14}\theta_{43} & 0.22\,\theta_{14} & 0.31 \end{pmatrix}$$

For $\langle\theta_{43}\rangle \sim \frac{1}{2}, \;\; \langle\theta_{14}\rangle \sim \frac{1}{10} \rightarrow$

Reasonable hierarchy and CKM-mixing

▲▼ Charged Leptons ▲▼

Major problem in **SU(5)**: Wrong mass relations:

$$m_s = m_\mu \;\&\; m_d = m_e \quad \text{at } M_{GUT}$$

SOLUTION:

▲▼**Splitting of $SU(5)$-reps** via the **FLUX mechanism**

Two types of fluxes:

▲M_{10}, M_5: connected to $U(1)$'s$\in SU(5)_\perp$: determine the chirality of complete $10, 5 \in SU(5)$.

▲N_Y: related to Cartan generators of $SU(5)_{GUT}$. They are taken along $U(1)_Y \in SU(5)_{GUT}$ and **split** $SU(5)$-reps.

$U(1)_\perp$–Flux on SM reps $\in$ **10**'s:

$$n_{(3,2)_{\frac{1}{6}}} - n_{(\bar{3},2)_{-\frac{1}{6}}} = M_{10} \tag{4}$$

$$n_{(\bar{3},1)_{-\frac{2}{3}}} - n_{(3,1)_{\frac{2}{3}}} = M_{10}$$

$$n_{(1,1)_1} - n_{(1,1)_{-1}} = M_{10} \tag{5}$$

$U(1)_\perp$– Flux on SM reps $\in$ **5**'s:

$$n_{(3,1)_{-\frac{1}{3}}} - n_{(\bar{3},1)_{\frac{1}{3}}} = M_5 \tag{6}$$

$$n_{(1,2)_{\frac{1}{2}}} - n_{(1,2)_{-\frac{1}{2}}} = M_5$$

(*...subject to:* $\sum_i M_{10}^i + \sum_j M_5^j = 0$)

$U(1)_Y-$**Flux**-splitting of **10**'s:

$$n_{(3,2)_{\frac{1}{6}}} - n_{(\bar{3},2)_{-\frac{1}{6}}} = M_{10}$$

$$n_{(\bar{3},1)_{-\frac{2}{3}}} - n_{(3,1)_{\frac{2}{3}}} = M_{10} - \mathbf{N_Y}$$

$$n_{(1,1)_1} - n_{(1,1)_{-1}} = M_{10} + \mathbf{N_Y}$$

$U(1)_Y-$ **Flux**-splitting of **5**'s:

$$n_{(3,1)_{-\frac{1}{3}}} - n_{(\bar{3},1)_{\frac{1}{3}}} = M_5$$

$$n_{(1,2)_{\frac{1}{2}}} - n_{(1,2)_{-\frac{1}{2}}} = M_5 + \mathbf{N_Y}$$

Application: Choice of M_{10_i}, M_{5_j} so that:

$$matter\,curve\,10^{(3)}\;:\quad\rightarrow\quad Q_2+u_2^c\equiv\begin{pmatrix}c\\ s\end{pmatrix}+c^c$$

$$matter\,curve\,10^{(4)}\;:\quad\rightarrow\quad e_2^c\equiv\mu^c$$

$(...plus\;\;extra\;\;vector\text{-}like\;\;u^c+\bar{u}^c)$

$$\Rightarrow\;M_{\ell epton}\neq M_{down}$$

$$M_\ell\;=\;\begin{pmatrix}\theta_{14}^2\theta_{43}^2 & \theta_{15}\theta_{14}\theta_{43} & \theta_{14}\,\theta_{43}\\ \theta_{14}^2\theta_{43} & \theta_{15}\theta_{43} & \theta_{14}\\ \theta_{14}\theta_{43} & \theta_{15} & 0.295\end{pmatrix}$$

$\rightarrow\;m_\mu\sim3\cdot m_s$!, $m_d\sim m_e$?...

▲▼Neutrinos ▲▼

Z_2-monodromy $\theta'_{12} \leftrightarrow \theta'_{21} \Rightarrow$

$$\theta'_{12} = \textbf{Right-Handed Majorana Neutrino}$$

(*Bouchard et al, 0904.1419*)

Assume also:

$$\theta'_{14},\, \theta'_{41},\, \theta'_{14},\, \theta'_{41}$$

States are

$$\nu_{@} \approx \frac{1}{\sqrt{2}} \left[\nu_3 - \nu_2 + \sqrt{2}\langle\theta_{14}\rangle(\langle\theta_{43}\rangle - 1))\nu_1 \right]$$

$$\nu_{\odot} \approx \frac{1}{N} \left[\nu_3 + \nu_2 + \sqrt{2}\langle\theta_{14}\rangle(1 + \langle\theta_{43}\rangle)\nu_1 \right]$$

Parameters can be adjusted to give bi-tri maximal mixing.

SUMMARY

1. F-theory offers new insights in **Yukawa** textures
2. **Matter** localised on "curves" in **Internal Geometry**

   Chirality related to topology and to the internal flux
3. Wave functions localised along these "matter curves"
4. Yukawa couplings, at **triple intersections**
5. **Fermion mass textures** completely determined from
   a) $\lambda(q, a_G)$ and,
   b) familon vevs $\langle\theta_{ij}\rangle$
6. Flux mechanism:
   i) $m_\mu - m_s$ splitting at M_{GUT}
   ii) Doublet-Triplet splitting (*suppressing Proton Decay...*)